QUANTUM UNIVERSE: GEOMETRY & TOPOLOGY.
FINAL EXAM 2013/14

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\+ **Problem 1.** Prove that for all $p \geqslant 1$, $q \geqslant 1$ one obtains a tensor by contracting the kth upper with ℓth lower index in a tensor field of type (p, q) on a smooth real manifold.

\- **Problem 2.** Describe the set of all geodesic curves on the surface $\Sigma^2 = \{x^2 + y^2 = 1,\ z^2 + w^2 = 1\} \subset \mathbb{E}^4$, where x, y, z, w are Cartesian coordinates. Isometry? θ, φ?

\- **Problem 3.** Can it (always*) be that the sum of angles in a triangle formed by geodesics on a sphere $\mathbb{S}^2 \subset \mathbb{E}^3$ is greater than π?

\- • Can it (always*) be that the sum of angles in a triangle formed by geodesics on the Lobachevsky plane $\mathbb{H}^2$ is less than π?

\+ **Problem 4.** Show that [any interval in] the curve $r = \text{const} > 0$ is not a geodesic with respect to the parametrisation $d\ell^2 = dr^2 + \sinh^2 r\, d\varphi^2$ of the Lobachevsky plane.

\- **Problem 5.** Prove that the scalar curvature R of a two-dimensional real Riemannian manifold M^2 with a symmetric Riemannian connection associated with the metric $g_{\mu\nu}$ is related to just one component of the Riemann tensor $R_{ij,k\ell}$ on M^2 by the formula

$$R = \frac{2R_{12,12}}{\det(g_{\mu\nu})}.$$

Date: April 4, 2014.
Do not postpone your success until May Day. GOOD LUCK!

$\frac{\partial}{\partial x} \sinh x = \frac{\partial}{\partial x} \frac{e^x - e^{-x}}{2} = \frac{e^x + e^{-x}}{2}$

$g = \sum A^{ij} g_{ij}$

$g^{ij} = \frac{A^{ji}}{g}$